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STRUCTURAL DESIGN OF BRICK MASONRY ARCHES

INTRODUCTION

The railway bridge at Maidenhead, England, constructed in 1838, is a brick arch with a span of 128 ft and a rise of 24.3 ft. This arch was designed by engineer Marc Brunel, who is also credited with being the first to use reinforced brick masonry. A similar brick arch railway bridge was constructed on North Avenue, Baltimore, Maryland, in 1895. It has a span of 130 ft and a rise of 26 ft. These outstanding examples are cited only to illustrate the structural capabilities of the brick arch—capabilities on which designers may rely when architectural or structural considerations suggest their use in modern design.

This issue of *Technical Notes* covers the structural design of major and minor brick masonry arches.

Minor arches are those whose spans do not exceed 6 ft and with maximum rise-to-span ratios of 0.15. Coefficients are given from which the horizontal thrust of such arches may be determined. Equations are presented for obtaining compressive stresses developed in the masonry and for determining stability against sliding.

Derivation of thrust coefficients and equations are based on the hypothesis of least crown thrust, as described in *Technical Notes*, No. 31, and the following assumptions have been made:

1. The thrust at the crown is horizontal and passes through the upper ⅓ point of the arch.
2. The reaction passes through the lower ⅓ point of the arch at the skewback.
3. The equilibrium polygon lies completely within the middle ⅓ of the arch.

Figure 1 illustrates jack and segmental arches.

Major arches are those with spans in excess of 6 ft or rise-to-span ratios greater than 0.15. In this issue of *Technical Notes* an example is given of major arch design based on the equations for redundant moments and forces presented in the publication, "Frames and Arches"[1]. The method of analysis presented in this book is substantially shorter than others in current use.

[1] "Frames and Arches," by Valerian Leontovich, McGraw Hill Book Co., 1959.

MINOR ARCH LOADING

The loads falling upon a minor arch may consist of live loads and dead loads from floors, roofs, walls and other structural members. These are applied as point loads or as uniform loads fully or partially distributed. A method of determining imposed loads on a member spanning small openings is described in *Technical Notes*, No. 17H. A brief resume of that explanation is given here.

The dead load of a wall above an arch may be assumed to be the weight of wall contained within a triangle immediately above the opening. The sides of this triangle are at 45-deg angles to the base. Therefore, its height is ½ of the span. Such triangular loading may be assumed to be equivalent to a uniformly distributed load of 1⅓ times the triangular load.

Superimposed uniform loads above this triangle may be carried by arching action of the masonry wall itself. Uniform live and dead loads occurring below the apex of the triangle are ap-

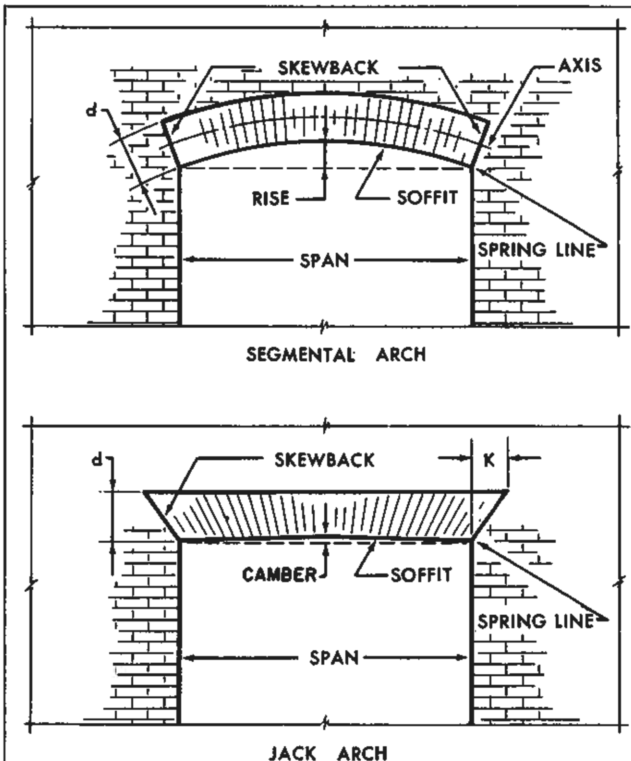

FIG. 1

*Originally published in October 1967, this Technical Notes has been reviewed and reissued with editorial changes.

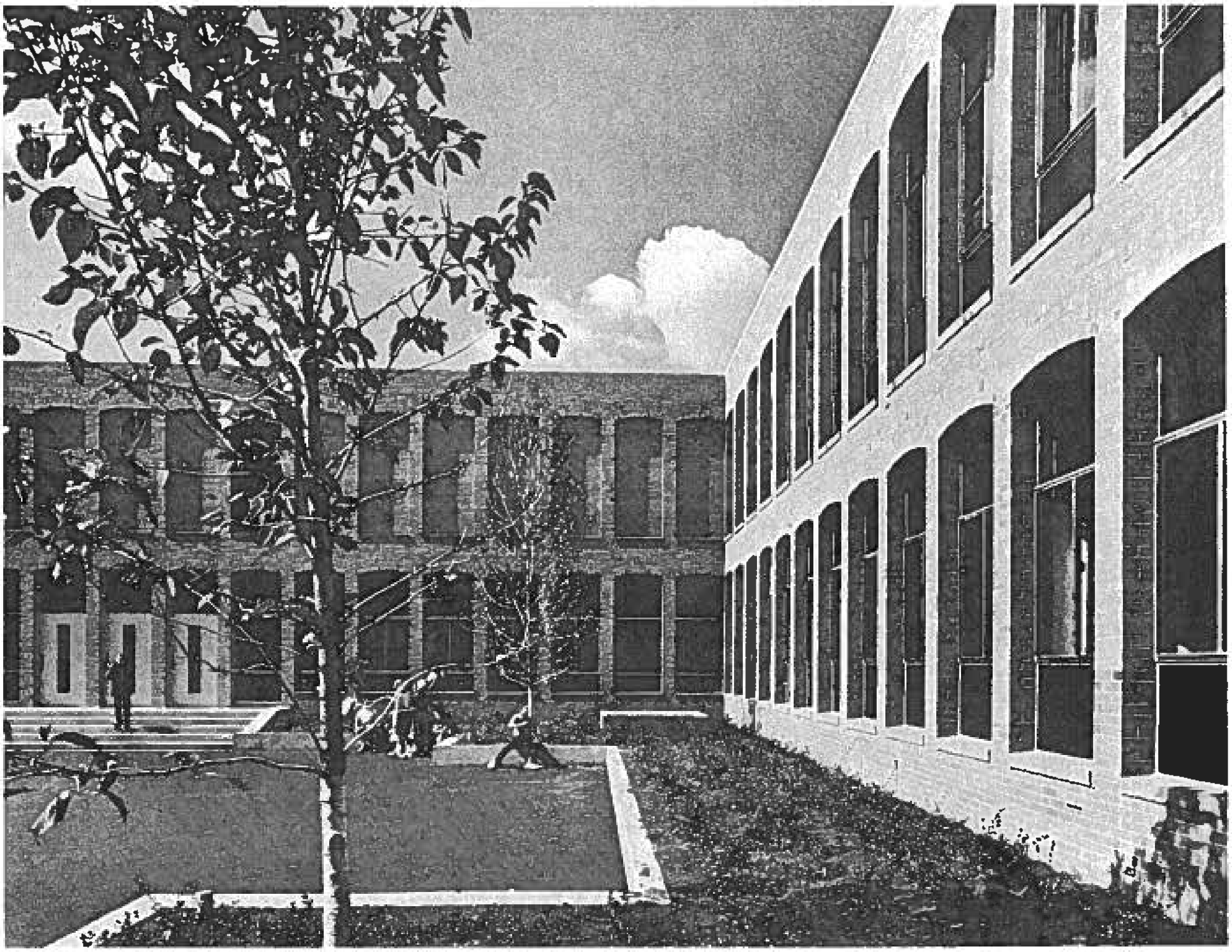

FIG. 2
High School in Columbus, Indiana
Harry Weese & Associates, Architects

plied directly upon the arch for design purposes.

Heavy concentrated loads should not be allowed to bear directly on minor arches. This is especially true of jack arches. Minor concentrated loads bearing on, or nearly directly on, the arch may safely be assumed to be equivalent to a uniformly distributed load equal to twice the concentrated load.

Figure 2 shows the use of minor arches in contemporary architecture.

MINOR ARCH DESIGN

There are three methods of failure of unreinforced masonry arches: (a) by rotation of one section of the arch about the edge of a joint; (b) by the sliding of one section of the arch on another or on the skewback; (c) by crushing of the masonry.

(a) Rotation. The assumption for the design of minor arches, that the equilibrium polygon lies entirely within the middle third of the arch section, precludes the rotation of one section of the arch about the edge of a joint or the development of tensile stresses in either the intrados or extrados.

(b) Sliding. The coefficient of friction between the units composing a brick or tile masonry arch is at least 0.60, without considering the additional resistance to sliding resulting from bond between mortar and the masonry units. This corresponds to an angle of friction of approximately 31 deg. If that angle, which the line of resistance of the arch makes with the normal to the joint between arch sections, is less than the angle of friction, the arch is stable against sliding. This angle can be determined graphically, as illustrated in *Technical Notes*, No. 31, or may be determined mathematically by the following formula:

$$\beta = \tan^{-1}\left(\frac{W}{2H} - \gamma\right) \qquad (1)$$

where:

β = angle between line of resistance and normal to joint,

W = total equivalent uniform load on arch,

H = crown thrust and

γ = angle of joint with vertical.

For minor segmental arches, the angle between the line of resistance and the normal to the joint is greatest at the skewback. This will also be true for jack arches if the joints are radial about a center at the intersection of the planes of the skewbacks. However, if the joints are not radial about this center, each joint should be investigated for resistance to sliding. This can be done most easily by constructing an equilibrium polygon, assuming that the crown thrust is applied at the top of the middle third and the reaction at the skewback is applied at the bottom of the middle third of the section.

For segmental arches with radial joints, the angle (γ) between the skewback and the vertical is

$$\gamma = \tan^{-1} \frac{4rS}{S^2 - 4r^2} \quad (2)$$

or in terms of the radius of curvature

$$\gamma = \sin^{-1} \frac{S}{2R} \quad (3)$$

For jack arches in which the skewback equals ½ in. per ft of span for each 4 in. of arch depth, the angle (γ) that the skewback makes with the vertical is

$$\gamma = \tan^{-1} \frac{S}{8} \quad (4)$$

In equations 2, 3 and 4:
S = span,
r = rise,
R = radius of curvature.

(c) Crushing.

(1) Segmental Arch. Figure 3 is a graphic representation of thrust coefficients (H/W) for segmental arches subjected to uniform load over the entire span. Once the thrust coefficient is determined for a particular arch, the horizontal thrust (H) may be determined as the product of the thrust coefficient and the total load (W). To determine the proper thrust coefficient, one must first determine the characteristics of the arch, S/r and S/d:

where:
S = the clear span,
r = the rise of the soffit and
d = the depth of the arch.

In these ratios and in the ratios and equations that follow, all terms of length must be expressed in the same units; for example, in computing S/r and S/d, if S is in feet, r and d must be in feet also.

If the applied load is triangular or concentrated, the above method may still be used, but the horizontal thrust coefficient must be increased by ⅓ for triangular loading and doubled for concentrated loads.

Once the horizontal thrust has been determined, the maximum compressive stress in the masonry is determined by the following formula:

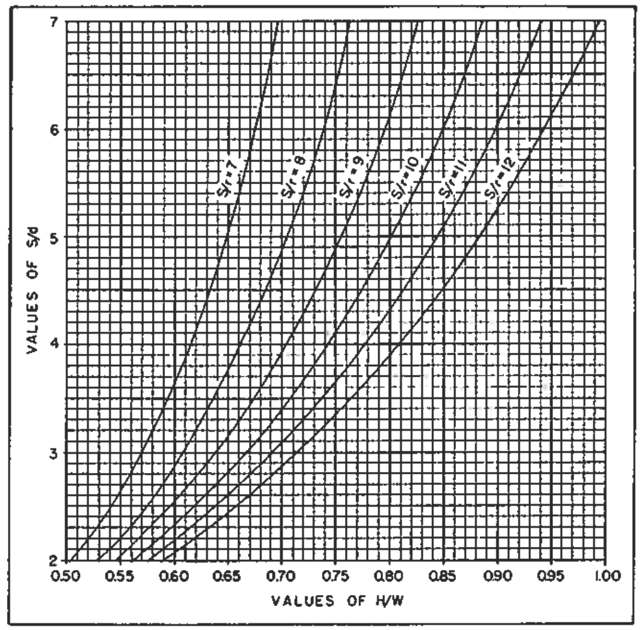

FIG. 3
Thrust Coefficients for Segmental Arches

$$f_m = \frac{2H}{bd} \quad (5)$$

In this equation:
f_m = maximum compressive stress in the arch in pounds per square inch,
H = horizontal thrust in pounds,
b = breadth of the arch in inches and
d = depth of the arch in inches.

This value is twice an axial compressive stress on the arch, due to a load H, because the horizontal thrust is located at the third point of the arch depth.

(2) Jack Arch. The common rule for jack arches is to provide a skewback (K, measured horizontally) of ½ in. per ft of span for each 4 in. of arch depth. Jack arches are commonly constructed in depths of 8 and 12 in. with a camber of ⅛ in. per ft of span.

For jack arches, applying the same assumptions as previously outlined, the horizontal thrust at the spring line may be determined by the following formulae:

For uniform loading over full span,

$$H = \frac{3WS}{8d} \quad (6)$$

For triangular loading over full span,

$$H = \frac{WS}{2d} \quad (7)$$

Maximum compressive stress (f_m) in the jack arch may be determined from the following formulae:

$$f_m = \frac{2H}{bd} \quad (8)$$

The maximum compressive stress in a jack arch may be computed directly from the following formulae:

For uniform loading over full span,

$$f_m = \frac{3WS}{4bd^2} \quad (9)$$

For triangular loading,

$$f_m = \frac{WS}{bd^2} \quad (10)$$

Formulae 8, 9 and 10 include a factor which allows for non-axial loading. In formulae 6 through 10, inclusive:

H = horizontal thrust in pounds,
W = total load in pounds,
S = clear span in inches,
d = depth of arch in inches and
b = breadth of arch in inches.

THRUST RESISTANCE

Resistance to horizontal thrust, developed by the arch, is provided by the adjacent mass of masonry. In areas where limited masonry is available, i.e. corners, openings, etc., it may be necessary to check the resistance of the wall to the horizontal thrusts. Figure 4 illustrates how such resistance may be calculated.

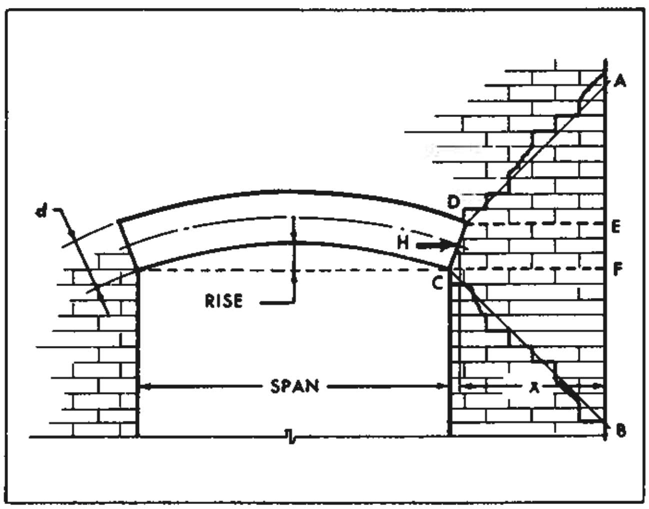

FIG. 4

It is assumed that the thrust of the arch attempts to move a volume of masonry enclosed by the boundary lines ABCD. For calculating purposes the area CDEF is equivalent in resistance. It can be seen that the thrust is acting against two planes of resistance, CF and DE. The resistance to arch thrust is determined by the following formula:

$$H_t = v_m nxt \quad (11)$$

By using the principle given in formula (11), the minimum distance from a corner or opening at which an arch may be located is easily determined. This can be done by writing formula (11) to solve for x, substituting actual arch thrust for resisting thrust:

$$x = \frac{H}{v_m nt} \quad (11a)$$

In these formulae:

H_t = resisting thrust in pounds,
v_m = allowable shearing stress in the masonry wall in pounds per square inch,
n = the number of resisting shear planes,
x = the distance from the center of the skewback to the end of the wall in inches and
t = wall thickness in inches.

The tendency for arch thrust to overturn a section of masonry, rather than slide it or rack it, must also be investigated. In general, such overturning forces are found to govern only at arches near the top of a wall, since that portion of masonry which tends to overturn must first become separated from the body of the wall.

ALLOWABLE STRESSES

Recommended allowable compressive stresses for use in the design of brick arches are given in Table 1. Recommended allowable shearing stresses in unreinforced walls for use in the design of abutments are given in Table 2. These are based on the requirements of *Recommended Building Code Requirements for Engineered Brick Masonry*, SCPI, May 1966.

TABLE 1

Allowable Compressive Stresses for Brick Masonry, psi [1]

Compressive Strength of Units, psi [2]	Mortar Type and Mix (parts by volume)		
	Type M 1PC:1/4L:3S	Type S 1PC:1/2L:4-1/2S	Type N 1PC:1L:6S
14,000 plus	1150	975	800
12,000	1000	850	700
10,000	850	725	600
8,000	700	600	500
6,000	550	475	400
4,000	400	350	300
2,000	250	225	200

[1]Based on *Recommended Building Code Requirements for Engineered Brick Masonry*, SCPI, May 1966.
[2]Linear interpolation is permissible.

TABLE 2

Recommended Allowable Shearing Stresses in Unreinforced Brick Masonry Walls

Mortar Type	Allowable Shearing Stress
M or S	50
N	40

MAJOR ARCH LOADING

The principal forces acting upon arches in buildings are the result of vertical dead and live loads and wind loads. Many masonry arches are integral with surrounding masonry. In such instances, loads transmitted to the arch through the masonry are indeterminate, due to arching action of adjacent masonry.

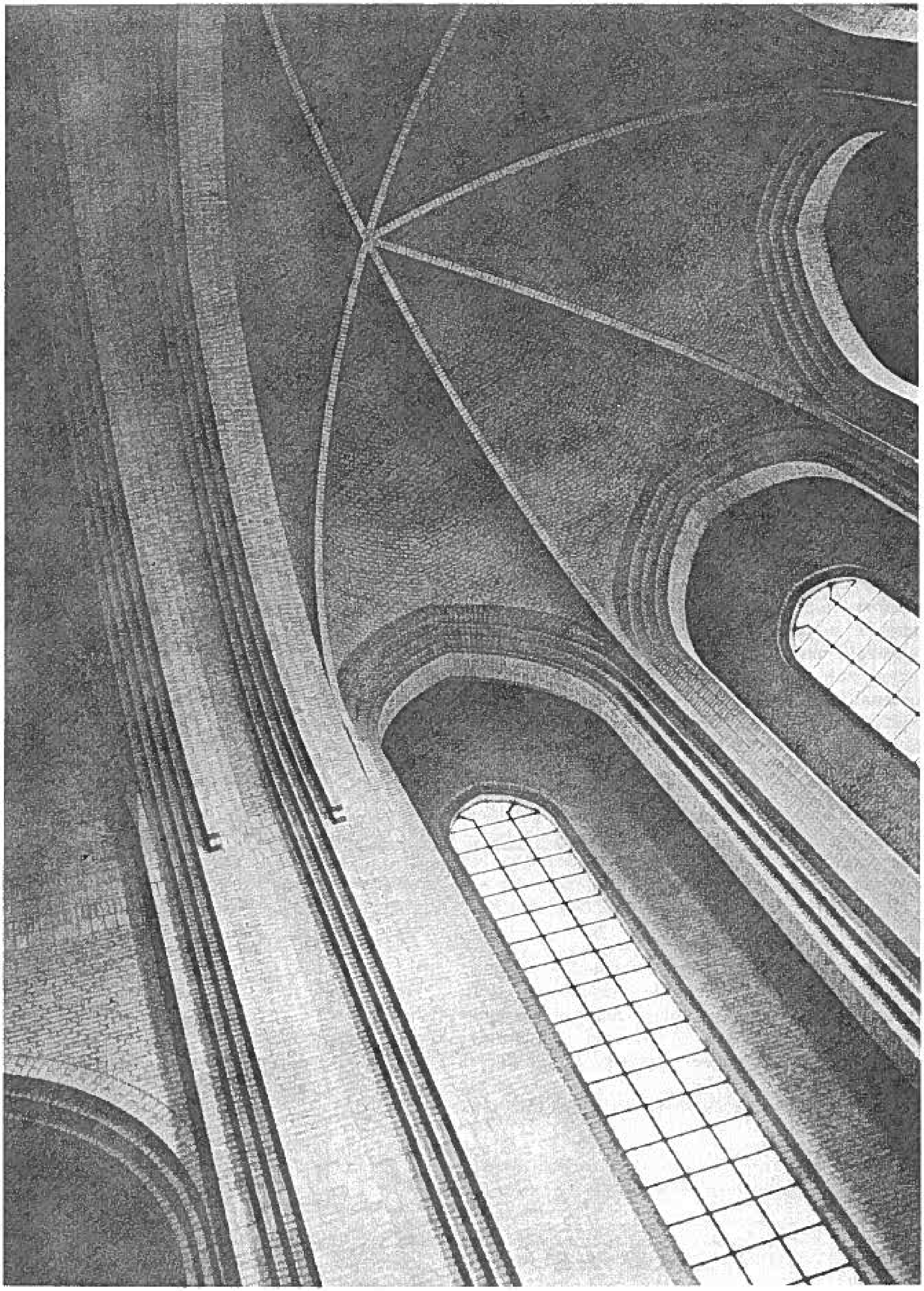

FIG. 5
Grundtvig Church, Copenhagen, Denmark
Designed in 1913 by P. V. Jensen Klint
It was completed in 1940.

It is often assumed that the entire weight of masonry, above the soffit, presses vertically upon the arch. This certainly is not accurate, since even with dry masonry a part of the wall will be self-supporting. However, this assumption is certainly on the safe side. The passive resistance of the adjacent masonry materially affects the stability of an arch.

The designer must rely on empirical formulae, based on the performance of existing structures, to determine the loads on an arch. The dead load of masonry wall supported by an integral arch depends upon the arch rise and span and the wall height above the arch. It may be considered to be either uniform (rectangular) or variable (complementary parabolic) in distribution, or a combination thereof.

"Frames and Arches" gives solutions for arches with rise-to-span ratios (f/L) ranging from 0.0 to 0.6. The following recommended assumptions for loading of such arches are believed to be safe:

For low rise arches, $f/L = 0.2$ or less, a uniform load may be assumed. This load will be the weight of wall above the crown of the arch up to a maximum height of L/4.

For higher rise arches a dead load consisting of uniform plus complementary parabolic loading may be assumed. The maximum ordinate of the parabolic loading will be equal to a weight of wall whose height is the rise of the arch. The minimum ordinate of the parabolic loading will be zero. The uniform loading will be the weight of the wall above the crown of the arch up to a maximum height of $L^2/100$.

Uniform floor and roof loads are applied as a uniform load on the arch. Small concentrated loads may be treated as uniform loads of twice the magnitude. Large concentrated loads may be treated as point loads on the arch.

Several major arches are shown in Fig. 5.

MAJOR ARCH DESIGN

General. "Frames and Arches" provides straightforward equations by which redundant moments and forces in arched members may be determined. The reader is referred to a discussion of this book which appears in *Technical Notes*, No. 31.

Without repeating the forementioned discussion here, let it suffice to say that, for relatively high-rise ($f/L = 0.2$) constant-section arches, Method A of Section 22 yields the proper solutions. The recommendations for use of this section are:

1. Establish principal dimensions of the arch.
2. On this basis and depending upon the established shape and f/L ratio of the arch, obtain the corresponding k value of the arch (see Table 3).
3. Obtain the elastic parameters (α, β, γ and δ), load constants and general constants.
4. Perform the algebraic operations with the given equations.

Equations. The equations are based upon a horizontal and vertical grid coordinate system with origin at the intersection of the arch axis and left skewback. Distances x and y are coordinates of the arch axis. The general equation for the parabolic arch axis is:

$$y = 4f\left(1 - \frac{x}{L}\right)\frac{x}{L}$$

Each set of equations depends upon the loading conditions. Among the solutions included with those in Section 22, Method A are the following: For vertical complementary parabolic loading:

$$M_1 = M_2 = \frac{WL}{F}(JS - 2T)$$

$$H_1 = H_2 = \frac{WL}{Ff}(K - 2JT)$$

$$V_1 = V_2 = W/2$$

When $x \leqq L/2$:

$$M_x = M_1 + \frac{WL}{16}\left[1 - \left(\frac{L-2x}{L}\right)^4\right] - H_1 y$$

$$N_x = \frac{W}{2}\left(\frac{L-2x}{L}\right)^3 \sin\phi + H_1\cos\phi$$

$$Q_x = \frac{W}{2}\left(\frac{L-2x}{L}\right)^3 \cos\phi + H_1\sin\phi$$

For vertical uniform load over the entire arch:

M and Q are zero at any section of the arch.

$$H_1 = H_2 = \frac{WL}{Ff}(K - 2JT)$$

$$V_1 = V_2 = W/2$$

TABLE 3

Values of k

Arch rise-to span-ratio f/L	0.2	0.3	0.4	0.5	0.6
Arch k value	1.28	1.56	1.90	2.40	2.80

Note: Adapted from Table 12, "Frames and Arches."

TABLE 4

Values of ϕ

Arch ratio f/L	Values of ϕ where x =					
	0 and L	0.1L and 0.9L	0.2L and 0.8L	0.3L and 0.7L	0.4L and 0.6L	0.5L
0.20	38°40′	32°37′	25°38′	17°45′	9°05′	0
0.30	50°12′	43°50′	35°45′	25°38′	13°30′	0
0.40	58°00′	52°00′	43°50′	32°37′	17°45′	0
0.50	63°26′	58°00′	50°12′	38°40′	21°48′	0
0.60	67°23′	62°29′	55°13′	43°50′	25°38′	0

Note: From Table 10, "Frames and Arches."

TABLE 5
Values of Load Constants S and T

Arch k Value		Uniform load over the entire span	Complementary parabolic load over the entire span
3.00	S	1.0290	0.6095
	T	0.7000	0.4429
2.00	S	0.9143	0.5333
	T	0.6000	0.3714
1.40	S	0.8457	0.4876
	T	0.5400	0.3286
1.30	S	0.8343	0.4800
	T	0.5300	0.3214
1.20	S	0.8229	0.4724
	T	0.5200	0.3143

Note: From Table 18, "Frames and Arches." Intermediate values may be obtained by interpolation.

TABLE 6
Arch Parameters α, β, γ and δ

Arch k Value	α	β	γ	δ
3.00	7.20	2.80	8.229	2.971
2.00	5.60	2.40	7.314	2.286
1.80	5.28	2.32	7.131	2.149
1.60	4.96	2.24	6.949	2.011
1.40	4.64	2.16	6.766	1.874
1.20	4.32	2.08	6.583	1.737

Note: From Table 13, "Frames and Arches." Intermediate values may be obtained by interpolation.

When $x \leqq L/2$:

$$N_x = H_1\cos\phi + \frac{W}{2}\left(1 - \frac{2x}{L}\right)\sin\phi$$

"Frames and Arches" also contains equations for other loading conditions; e.g. concentrated loads.

Notation. In these equations, the subscripts 1 and 2 denote the left and right supports respectively. The subscript x denotes values at any horizontal distance, x, from the origin. ϕ is the angle, at any point, whose tangent is the slope of the arch axis at that point. (See Table 4.)

M = moment
N = axial force
Q = shearing force
f = rise of the arch
W = total load under consideration
H = horizontal thrust
V = vertical reaction
L = span of the arch

S and T are load constants (see Table 5).
J, F and K are constants, determined by:

$J = 1 + (\delta/\gamma)$ $\quad$ $F = \theta - \gamma J^2$

$K = S\theta/\gamma$

α, β, γ and δ are parameters (see Table 6).

$\theta = 2(\alpha + \beta)$

ILLUSTRATIVE EXAMPLE

Problem. Using the equations given in the book, "Frames and Arches," design a parabolic brick masonry arch to meet the following requirements. The arch is integral with a loadbearing, brick-and-brick cavity wall. Wall weight is 80 psf. The arch dimensions are: span, 20 ft; rise, 12 ft; depth, 16 in.; thickness, 12 in.; total wall height, 8 ft. The uniform roof load, bearing on the wall above the arch, is 1200 lb per ft. The arch is of solid brick (4000 psi) and type N mortar; allowable compressive stress in the arch is 300 psi.

Solution. The arch is a constant-section, high-rise, symmetrical, parabolic, hingeless arch; therefore, the equations previously given in this issue of *Technical Notes* are applicable. Each different loading condition must be analyzed separately. Similar loads, e.g. all uniform loads, may be added and treated as a single load. Moments, shears and thrusts resulting from each loading condition are combined to give total values. For symmetrically loaded symmetrical arches, only ½ of the arch need be analyzed.

The loads carried by the arch are:

Uniform loads
- Wall dead load
  - $(80)(20^2/100)$ = 320 lb per ft
- Roof dead + live load = 1200
- Arch dead load in excess of wall weight (approx.) = 260
- Total uniform load = 1780 lb per ft
- $W = (1780)(20) = 35{,}600$ lb

Complementary parabolic loading
- Maximum ordinate
  - $p = (80)(12)$ = 960 lb per ft
- Minimum ordinate = 0
- $W = pL/3 = 960(20)/3$ = 6400 lb

Numbers before the following paragraphs refer to the outline of the recommended sequence.

1. The principal arch dimensions are:
   $d = 16$ in. $\quad t = 12$ in. $\quad L = 20$ ft
   $f = 12$ ft $\quad f/L = 0.6$
2. From Table 3, $k = 2.80$.

3a. For parabolic loading:
From Table 5,
$S = 0.5943$ $\quad T = 0.4286$
From Table 6,
$\alpha = 6.88$ $\quad \gamma = 8.046$
$\beta = 2.72$ $\quad \delta = 2.834$
From the given relationships,
$J = 1.3522$ $\quad F = 4.4884$
$\theta = 19.20$ $\quad K = 1.4182$

3b. For vertical uniform loads:
From Table 5,
$S = 1.0061$ $\quad T = 0.6800$
From Table 6 (note that α, β, γ and δ are the same for any given arch dimensions),
$\alpha = 6.88$ $\quad \gamma = 8.046$
$\beta = 2.72$ $\quad \delta = 2.834$
From the given relationships:
$J = 1.3522$ $\quad F = 4.4884$
$\theta = 19.20$ $\quad K = 2.4008$

4. The necessary substitution may now be made to evaluate the design moments and forces. In

TABLE 7

Values of M_x, N_x and Q_x at Increments of x = 0.1L

x (ft)	0.0	2.0	4.0	6.0	8.0	10.0
Vertical Complementary Parabolic Loading:						
M_x	−1528.3	+534.9	+706.0	+60.2	−634.5	−917.3
N_x	3190.7	1737.5	919.0	586.0	566.2	615.8
Q_x	+662.2	+210.9	−111.4	−278.7	−243.3	0.0
Vertical Uniform Loading:						
N_x	19,300	16,100	13,000	10,300	8200	7430

Note: The above moments are in foot-pounds. Shears and axial thrusts are in pounds.

this example, moments, shears and axial thrusts are determined at increments of 0.1L (each 2 ft of span). Tabular computations are suggested for ease in evaluating these equations. The results of such tabular computations are shown in Table 7.

The stresses in the arch may be determined from the following equations:

$$f_m \leq \frac{N_x}{td} \pm \frac{6M_x}{td^2} \qquad (12)$$

$$v_m = \frac{Q_x}{td} \qquad (13)$$

In the above equations, f_m denotes maximum and minimum fiber stresses and v_m denotes shearing stresses. All quantities in the equations must be in units of inches and pounds. Table 8 shows stresses in the arch.

Plus signs indicate compression and minus signs tension, for values of f_m. These signs have only directional significance for values of v_m. No tensile stresses should be permitted in unreinforced masonry arches under static loading conditions. The reader is referred to *Technical Notes*, No. 31 for a discussion of mortars in arch construction. See Table 1 for allowable compressive stresses in brick masonry.

TABLE 8

Stresses in the Arch at Increments of x = 0.1L

x (ft)	0.0	2.0	4.0	6.0	8.0	10.0
Max f_m	+152.8	+105.5	+89.0	+57.9	+60.5	+63.3
Min f_m	+81.2	+ 80.5	+56.0	+55.1	+30.9	+20.3
v_m	+ 3.45	+ 1.10	−0.58	−1.45	−1.27	0.00

Note: The above stresses are in pounds per square inch.

Interpretation of Results. Table 8 indicates that the compressive stresses are, in all instances, less than the allowable 300 psi. No tensile stresses exist. The shearing stresses are insignificant. The arch is adequate. The moments and shears are caused by other than uniform loads. For this arch, and perhaps for most arches, the predominant load is uniformly distributed. As a result the moments and shears are relatively small and the arch is predominantly in compression.